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BHAGWAGAR

Solution by Azadan

Exam I, MTH 213 Discrete Math., Summer 2018

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Score = $\frac{\cancel{74}}{76}$

4/4 **QUESTION 1.** (a)(**5 points**) Use the 4-method to convince me that $\sqrt{46}$ is irrational

Assume $\sqrt{46}$ is rational; $\sqrt{46} = \frac{a}{b}$, for some $a, b \in Z$, $b \neq 0$, $gcd(a,b) = 1$

$46 = \frac{a^2}{b^2}$, $gcd(a^2, b^2) = 1$; 46 is even ∴ a^2 is even & b^2 is odd ∴ a is even, b is odd

$\sqrt{46} = \frac{2k}{2m+1}$; for some $k, m \in Z$; $46 = \frac{(2k)^2}{(2m+1)^2} = \frac{4k^2}{4m^2+4m+1}$; $184m^2 + 184m + 46 = 4k^2$; $46(m^2+m) + \frac{46}{4} = k^2$; non-integer ≠ integer, Contradiction, Assumption invalid ∴ $\sqrt{46}$ is irrational

(b)(**5 points**) Convince me that $\sqrt{65}$ is irrational

Assume $\sqrt{65}$ is rational; $\sqrt{65} = \frac{a}{b}$, for some $a, b \in Z$, $b \neq 0$, $gcd(a,b) = 1$;

$65 = \frac{a^2}{b^2}$, $gcd(a^2, b^2) = 1$; $b^2 \times 5 \times 13 = a^2$; $5 \times 13 \mid a^2 \Rightarrow 5 \times 13 \mid a$; $a = 5 \times 13 \times k$, for some $k \in Z$

$b^2 \times 5 \times 13 = (5 \times 13 \times k)^2$; $b^2 \times \cancel{5} \times \cancel{13} = 5^{\cancel{2}} \times 13^{\cancel{2}} \times k^2$; $5 \times 13 \mid b^2 \Rightarrow 5 \times 13 \mid b$;

implies

5×13 is a factor of both a and b, but $gcd(a,b) = 1$; Contradiction, Assumption invalid ∴ $\sqrt{65}$ is irrational //

5/5

(c) (**5 points**) Let m be an odd number. Prove that $m = 2k - 1$ for some integer $k \in Z$.

m is odd, $m = 2p + 1$, $p \in Z$; Let $p = k - 1$, $k \in Z$; $m = 2(k-1) + 1$;

$m = 2k - 2 + 1 = 2k - 1$ //

5/5

QUESTION 2. (a) (**5 points**) If possible find all solutions of $8x = 12$ over planet Z_{20}

$gcd(8, 20) = 4$, $4 \mid 12$ ✓ ∴ 4 distinct solutions; $q = \frac{n}{gcd(a,n)} = 5$;

$x_s = 4$; ∴ $x = \{4, 9, 14, 19\}$ //

5/5

(b) (**3 points**)In view of (a), find all integers, say x, in Planet Z that satisfy $8x \ (mod\ 20) = 12$

Since $q = 5$ and $x_s = 4$;

$x = \{4 + 5k \mid k \in Z\}$ //

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(c)(**5 points**) Find all integers, say x, in Planet Z that satisfy $3x \ (mod\ 11) = 10$

In Z_{11}, $3x = 10$; $gcd(3, 11) = 1$; $1 \mid 10$ ✓ ∴ 1 distinct solution;

$x = 7$; In Z, $x = \{7 + 11k \mid k \in Z\}$ //

5/5

(d)(**7 points)** The temperature of a city in Canada is x Celsius, where $-180 < x < 0$ (i.e., x is negative). Given $x \ (mod\ 9) = 4$, $x \ (mod\ 4) = 1$, and $x \ (mod\ 5) = 2$. Find the value of x.

$n_1 \quad \lambda_1 \quad n_2 \quad \lambda_2 \quad n_3 \quad \lambda_3$

gcd(9,4) = 1 ✓, gcd(9,5) = 1 ✓, gcd(4,5) = 1 ✓ ∴ CRT applies;
n = 180; m_1 = 20; In Z_9, $20x_1 = 1$; $2x_1 = 1$; $x_1 = \{5\}$;
m_2 = 45; In Z_4, $45x_2 = 1$; $x_2 = 1$; $x_2 = \{1\}$;
m_3 = 36; In Z_5, $36x_3 = 1$; $x_3 = 1$; $x_3 = \{1\}$;
$x_s = (m_1 n_1 \lambda_1 + m_2 n_2 \lambda_2 + m_3 n_3 \lambda_3) \bmod n = (400 + 45 + 72) \bmod 180 =$
$= 517 \bmod 180 = 157$;
$-180 < x < 0$ ∴ $x = 157 - 180 = -23$ //

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(e)(**5 points)** Let x be number of females in MTH 111. Given $x \ (mod\ 7) = 6$, $2x \ (mod\ 10) = 4$. Find all possible values of x, where $0 < x < 70$.(Hint: THINK!, it is not difficult)

$x \ (mod\ 7) = 6$; $x \ (mod\ 5) = 2$; gcd(7,5) = 1 ∴ CRT applies
n = 35; m_1 = 5; In Z_7, $5x_1 = 1$; $x_1 = \{3\}$;
m_2 = 7; In Z_5, $7x_2 = 1$; $x_2 = \{3\}$;
$x_s = (90 + 42) \bmod 35 = 132 \bmod 35 = 27$
x = 27 & 27 + 35 = 62
∴ x = {27, 62}

? $\frac{2x}{10}$ gives q + 4
$\frac{x}{5}$ gives q + 2

5/5

He gave Diff Method which is correct. However, All of you should know the METHOD I EXPLAINED in Class

(f) (**4 points)** Find $gcd(112, 175)$

175 mod 112 = 63; 112 mod 63 = 49; 63 mod 49 = 14;
49 mod 14 = 7; 14 mod 7 = 0; ∴ gcd(112,175) = 7 //

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(g) (**4 points)** Find $(146)_8 \times (54)_8$

$$\begin{array}{r} (146)_8 \\ \times (54)_8 \\ \hline (630)_8 \\ + (7760)_8 \\ \hline (10610)_8 \end{array}$$

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(i) (**4 points)** Convert 317 to base 16

317 div 16 = 19; 19 div 16 = 1; 1 div 16 = 0.
317 mod 16 = 13 = D; 19 mod 16 = 3; 1 mod 16 = 1;

∴ $(317)_{10} = (13D)_{16}$ //

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QUESTION 3. (a)(**4 points**) Let x, y, z be strings of binary codes, $x = 11011$, $y = 10110$, and $z = 01110$. Find $(x \oplus z) \vee y$

$x = 11011$
$z = 01110$
$y = 10110$
$(x \oplus z) \vee y = 10111$

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(b) (**6 points**) Let S_1, S_2, S_3 be some statements. Use truth table and convince me that

$$(S_1 \wedge S_2) \Rightarrow S_3 \equiv (S_1 \Rightarrow S_3) \vee (S_2 \Rightarrow S_3)$$

6/6 $(S_1 \wedge S_2) \Rightarrow S_3$ identical, ∴ $(S_1 \Rightarrow S_3) \vee (S_2 \Rightarrow S_3)$

S_1	S_2	S_3	$S_1 \wedge S_2$	$S_1 \Rightarrow S_3$	$S_2 \Rightarrow S_3$	$(S_1 \wedge S_2) \Rightarrow S_3$	$(S_1 \Rightarrow S_3) \vee (S_2 \Rightarrow S_3)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	0	1	0	1	1	1	1
1	0	0	0	0	1	1	1
0	1	1	0	1	1	1	1
0	1	0	0	1	0	1	1
0	0	1	0	1	1	1	1
0	0	0	0	1	1	1	1

(c) (**5 points**) Let S_1, S_2 be some statements. Is the expression $[(S_1 \Rightarrow S_2) \wedge \neg S_2] \Rightarrow \neg S_1$ a tautology? Explain using truth-table.

~~not identical ∴ Not a tautology~~

S_1	S_2	$S_1 \Rightarrow S_2$	$\neg S_2$	$(S_1 \Rightarrow S_2) \wedge \neg S_2$	$\neg S_1$	$[(S_1 \Rightarrow S_2) \wedge \neg S_2] \Rightarrow \neg S_1$
1	1	1	0	0	0	1
1	0	0	1	0	0	1
0	1	1	0	0	1	1
0	0	1	1	1	1	1

Tautology.

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(d) Write down T or F (**9 points**)

True

(i) $\exists!\, x \in N^*$ such that $\forall\, y \in Q$, we have $xy - 2y = 0$ T

(ii) $\forall\, x \in Q, \exists!\, y \in N^*$ such that $yx - 5x = 0$ F, if $x = 0$, then y is not unique.

(iii) $\forall\, y \in R^*, \exists!\, x \in R^*$ such that $yx - y^2 = 0$ T

(iv) If $\exists!\, x \in N^*$ such that $x^2 - 4x = 0$, then $4x - 1 = 12$ F (true, $x \neq 0$; false)

(v) If $\exists\, x \in Z$ such that $x^2 = 4$, then $x^3 = 8$ F, when $x = -2$, $x^3 = -8$, not 8. (True $x = \pm 2$; False)

(vi) If $\exists!\, x \in N^*$ and $\exists!\, y \in N$ such that $x^2 + y^2 = 1$, then $y - x = x - y$ F, $x = 1$, $y = 0$, $y - x = -1$, $x - y = 1$ ($x = 1$ true, $y = 0$; False)

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$x(x-4) = 0$